Wave Dynamics in the Channels of Variable Cross-Section

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Dynamics of long sea waves in the channels of variable depth and variable rectangular cross-section is discussed within various approximations – from the shallow water equations to those of nonlinear dispersion theory. General approach permitting to find traveling (non-reflective) waves in inhomogeneous channels is demonstrated within the framework of the shallow water linear theory. The appropriate conditions are determined by solving a system of ordinary differential equations. The so-called self-consistent channel in which the width is connected with its depth in a specific way is studied in detail. Within the linear theory of shallow water, a wave does not reflect from the bottom irregularities. The wave shape remains unchanged on the records of the wave gauges (mareographs) fixed along the channel axis, but it varies in space. Nonlinearity and dispersion lead to the wave transformation in such a channel. Within the framework of the shallow water weakly nonlinear theory, the wave shape is described by the Riemann solution, and the wave breaks (gradient catastrophe) quicker in the zones of decreasing depth. The modified Korteweg – de Vries equation describing evolution of a solitary wave of weak but finite amplitude in a self-consistent channel, the depth of which can vary arbitrary, is derived. Some examples of a solitary wave transformation in such a channel are analyzed (particularly, a soliton adiabatic transformation in the channel with the slowly varying parameters, and a solitary wave fission into the group of solitons after it has passed the zone where the depth changes abruptly. The obtained solutions extend the class of those represented earlier by S.F. Dotsenko and his colleagues.

Keywords: traveling long waves, channels of variable cross-section, shallow water equations, Korteweg – de Vries equation.

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1. Introduction

Propagation of long tsunami-like waves in narrow bays and straits may result in wave field intensification and abnormal runup. Such situations were observed, for instance, during the recent catastrophic tsunami at the Samoan Islands in 2009 [1, 2] and in Japan in 2011 [3]. Significant wave intensification also occurs along the submarine canyons, and this was pointed out during the Indian Ocean tsunami in 2004 [4].

In such cases wave dynamics can be described mathematically in channel approximation for the water flow characteristics averaged over the cross-section. One-dimensional equations obtained in this approximation speed up the calculations, especially when the parameters of approaching waves are unknown (as it often happens in practice) or a bathymetry of a channel is known with a poor accuracy. As a result, it is possible to obtain rapid assessment of wave parameters on the basis of a small number of parameters characterizing the problem.

This approach to the long wave modeling was the main one in S.F. Dotsenko's research of the recent years. On the basis of the results of this research he published several works in partnership [5–7]. We point out in particular the paper [6] on the propagation of long waves from the Sea of Marmara to the Black Sea through the Bosporus, which is important for the channel theory application. Channel theory, which demonstrated its effectiveness for solving real problems, turned out to be important also for explaining the abnormal wave intensification and runup in Pago-Pago Harbor during the 2009 tsunami at the islands of American Samoa [8].

It is possible to find accurate solutions describing nonlinear wave runup on the shores of narrow bays and straits within the framework of shallow water channel theory [9–11]. Moreover, it is shown that this theory explains the dynamics of freak waves on the shore and correlates well with their observations [12]. We point out in particular the fact that nonlinear waves in inclined channels with parabolic cross-section are non-reflective and do not lose energy during the propagation [13]. Mathematically, the validity of channel theory application for solving such problems was proved in [14]. From the practical point of view, its accuracy is assessed by a comparison with direct numerical solution of shallow water equations for waves in the bay of Alaska [15–17].

In the present paper, which is a tribute to S.F. Dotsenko, we would like to give several examples of channel theory application for describing the dynamics of wave processes in the channels with rectangular cross-section and variable depth and width. The case of so-called self-consistent channel, when within the framework of shallow water linear theory wave reflection from the bottom obstacles is absent, was selected.

2. Traveling waves in the channels with variable cross-section

Usually, when it comes to the waves in inhomogeneous media, it is meant energy losses for reflection from the bottom irregularities [18]. Restriction of wave propagation along the main channel axis provides localization of energy transfer (as there is no cylindrical attenuation and diffraction losses), but does not prevent backscattering if the channel depth and width vary arbitrarily. Nevertheless, for special channel geometry the wave will not reflect, and it can propagate over large distances with no energy losses. We demonstrate this using the example of propagation of linear long waves in a rectangular channel with varying cross-section (Fig. 1).

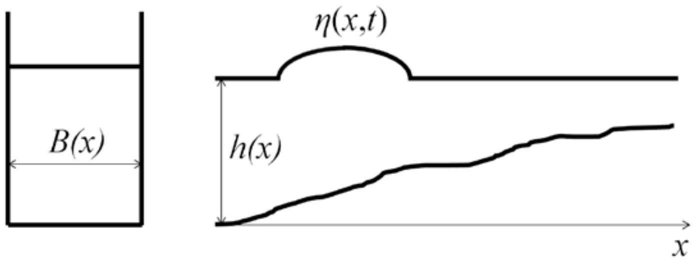

Fig. 1. Channel geometry: *on the left* – transversal projection, *on the right* – longitudinal projection

Initial equation for the analysis is the wave equation derived in [6, 19],

$$B(x)\frac{\partial^2\eta}{\partial t^2} - g\frac{\partial}{\partial x}\left(B(x)h(x)\frac{\partial\eta}{\partial x}\right) = 0\,, \tag{1}$$

where $h(x)$ is unperturbed depth; $B(x)$ is variable width of a channel; g is gravitational acceleration and $\eta(x, t)$ is vertical displacement of water surface.

The main idea in the search for solutions of wave equations in the form of traveling (non-reflective) waves comes down to the transformation of initial equation (1) with variable coefficients into wave-type equation with constant coefficients. For this purpose we perform the following substitute of water displacement (1):

$$\eta(x,t) = A(x)\Phi[t, \tau(x)], \tag{2}$$

where $A(x)$, $\Phi(t,\tau)$ and $\tau(x)$ are three unknown functions to be determined. Then the wave equation (1) transforms to Klein – Gordon equation with variable coefficients

$$AB\left[\frac{\partial^2\Phi}{\partial t^2} - gh\left(\frac{d\tau}{dx}\right)^2\frac{\partial^2\Phi}{\partial\tau^2}\right] - g\left[Bh\frac{dA}{dx}\frac{d\tau}{dx} + \frac{d}{dx}\left(BhA\frac{d\tau}{dx}\right)\right]\frac{\partial\Phi}{\partial\tau} - g\frac{d}{dx}\left(Bh\frac{dA}{dx}\right)\Phi = 0. \tag{3}$$

Let us consider conditions at which this equation will have constant coefficients. Wave operator in the first square bracket (d'Alembertian) will have constant coefficients if to assume that

$$gh\left(\frac{d\tau}{dx}\right)^2 = 1\,, \tag{4}$$

which determines the known relation for the time of wave propagation over uneven bottom

$$\tau(x) = \int\frac{dx}{c(x)} = \int\frac{dx}{\sqrt{gh(x)}}\,, \qquad c(x) = \sqrt{gh(x)}\,. \tag{5}$$

Here $c(x)$ is the wave celerity. In the second square bracket of equation (3) the term should be equal to zero (otherwise the dissipative term appears in the Klein-Gordon equation); this leads to a simple equation which can be integrated explicitly:

$$A(cB)^{1/2} \sim Ah^{1/4}B^{1/2} = \text{const}\,. \tag{6}$$

Relation (6) is the well-known Green's law for the waves in liquid with a smooth variation of a channel depth and width. However, in our case we do not impose conditions on the smoothness of channel characteristics variation.

In order that in the equation (3) all the coefficients to become constant, the last term should be proportional to AB:

$$g\frac{d}{dx}\left(Bh\frac{dA}{dx}\right) = ABP\,, \tag{7}$$

where P is arbitrary constant. As a result, equation (3) reduces to the Klein-Gordon equation with constant coefficients

$$\frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2 \Phi}{\partial \tau^2} - P\Phi = 0 , \tag{8}$$

and the existence of traveling waves within its framework becomes obvious.

After substituting expression (6), equation (7) becomes an ordinary differential equation for finding non-reflective configurations of the channel with rectangular cross-section:

$$\frac{d}{dx}\left[\sqrt{\frac{c}{B}}\frac{d}{dx}(Bc)\right] + 2\sqrt{\frac{B}{c}}P = 0 , \tag{9}$$

where the wave celerity $c(x) = \sqrt{gh(x)}$ is used instead of channel depth *h(x)* to simplify the recordings. As the equation (9) links two unknown functions $B(x)$ and $c(x)$ and, besides, two constants appear when integrating it, the number of non-reflective configurations of a channel is unlimited. The obtained configurations are very reasonable in terms of applicability of the shallow water theory: there are no singular solutions; the channel can be of unlimited length (these questions are discussed in [20]). In our opinion, this is exactly why a significant intensification of tsunami waves takes place in a series of cases mentioned in introduction.

If $P \neq 0$, then dispersive waves are the solution of the equation (8) and wave train can spread or, conversely, compress into a freak wave [21]. The dispersion here is related to the channel geometry, not to the known depth-related dispersion of waves on the water.

Let us consider the simplest channel configuration

$$B(x)c(x) = \sqrt{gh(x)}B(x) = \text{const} , \tag{10}$$

for which $P = 0$. The basin depth can vary arbitrarily, including abrupt (step-like) variations. We call such channel a *self-consistent* one. In this case a propagating wave is described by the simplest expression

$$\eta(x,t) = A_0 \Phi\left[t - \int_{x_0}^{x} \frac{dy}{\sqrt{gh(y)}}\right], \tag{11}$$

where A_0 is a constant amplitude of a wave and $\Phi(t)$ describes wave shape in a fixed point x_0. Thus, although the channel has a variable cross-section (its area is proportional to $h^{-1/2}$), wave amplitude remains constant and only the time of wave propagation along the channel varies. As a result, the wave records appear to be the same in different points of the channel, though the wave shape changes in space.

3. Dispersion and nonlinearity effect on waves in a self-consistent channel

As the wave shape (11) does not vary with the distance in the self-consisted channel (10) within the framework of shallow water linear theory, nonlinear and dispersive effects are accumulated. In the general case it is natural to expect that nonlinear and dispersive corrections will not be completely non-reflective, so that the process becomes rather complicated for analytical analysis. Nevertheless, if one assumes that the channel depth changes smoothly, then the reflection is certainly small and it is possible to derive evolutionary equation for the traveling wave using the asymptotic method. It was repeatedly quoted in literature, and we will quote it without derivation. It has the form of Korteweg – de Vries equation [22]:

$$\frac{\partial\eta}{\partial t}+c\left[1+\frac{3}{2}\frac{\eta}{h}\right]\frac{\partial\eta}{\partial x}+\frac{ch^2}{6}\frac{\partial^3\eta}{\partial x^3}+\sqrt{\frac{c}{B}}\frac{d}{dx}\left(\sqrt{cB}\right)\eta=0\,. \tag{12}$$

In the case of self-consistent channel, by virtue of the expression (10) the last term disappears from the equation (12) but Korteweg – de Vries equation remains the one with variable coefficients. As an example, we consider the transformation of a solitary wave (soliton) if the depth varies very smoothly. Soliton is described locally by the same expression as in the basin of constant depth:

$$\eta(x,t)=A\mathrm{sech}^2\left[\sqrt{\frac{3A}{4h}}\frac{x-c\left(1+\frac{A}{2h}\right)t}{h}\right] \tag{13}$$

with a characteristic width

$$\lambda\sim h\sqrt{\frac{h}{A}}\,. \tag{14}$$

Soliton amplitude is found from the law of conservation of energy because the reflection is absent:

$$E=B\int_{-\infty}^{+\infty}\eta^2(x,t)dx\sim B(x)A^2(x)\lambda(x)=\mathrm{const}\,. \tag{15}$$

For the self-consistent channel this results in the following law of soliton amplitude variation:

$$A(x)\sim h^{-2/3}\,, \tag{16}$$

which is weaker than in the channel of constant width $A(x)\sim h^{-1}$ [22]. According to the equation (14), soliton length is $\lambda(x)\sim h^{11/6}$.

Thus, the effect of dispersion and nonlinearity results in a traveling wave transformation, and if a soliton moves in shallow water its amplitude and length increase. Moreover, wave shape also changes due to the fact that the volume of water contained in a soliton (mass) varies with depth:

$$M=B\int_{-\infty}^{+\infty}\eta(x,t)dx\sim B(x)A(x)\lambda(x)\sim h^{2/3}\,, \tag{17}$$

while a gross mass must be constant. This means that a tail of positive polarity is formed behind the soliton if the wave propagates onshore and a tail of negative one – if it moves offshore. However, we will not dwell on this.

4. Soliton transformation on a bottom step

If the channel depth and width vary significantly, then the transition zone may be approximated by a bottom step. As the channel is a consistent one, in the linear theory of long waves the reflection from the bottom step is absent and the wave passes over it without deformation. Weak nonlinearity and dispersion have no time to “spoil” the process at such short distances. However, upon further propagation behind the bottom step in a channel of constant depth and width, nonlinearity and dispersion lead to the wave transformation. This can be described in detail within the framework of Korteweg – de Vries equation. This process in a channel of constant cross-section is discussed in many papers, and here we mention only [22]. Analytical approach is based on the following kinematic considerations. Let the channel depth before the step be h_1, and after it – h_2. Correspondingly, the channel width before the step is $B_1 \sim h_1^{-1/2}$, and after it – $B_2 = B_1 (h_1 / h_2)^{-1/2}$. Incoming soliton before the step (at a fixed time) is described by the expression (13)

$$\eta_1(x) = A_0 \mathrm{sech}^2\left[\sqrt{\frac{3A_0}{4h_1}}\frac{x}{h_1}\right]. \tag{18}$$

Wave amplitude does not change at the step in a self-consistent channel and the wavelength decreases proportionally to $(h_2 / h_1)^{1/2}$, because temporal duration remains the same during the transition from one layer to another. This means that after the bottom step and immediately prior to it the wave is described by the expression which is analogous to the expression (18) but with different duration:

$$\eta_2(x) = A_0 \mathrm{sech}^2\left[\sqrt{\frac{3A_0}{4h_2}}\frac{x}{h_1}\right], \tag{19}$$

and wavelength differs from the one that a soliton with the same amplitude should have after the step. As a result, the relationship between dispersion and nonlinearity, characterized by the Ursell number [22, 23]:

$$\mathrm{Ur} = \frac{A\lambda^2}{h^3}, \tag{20}$$

is violated. For the soliton (13) with the wavelength (14) Ursell parameter is equal to one. Now for the wave (19) Ursell parameter is

$$\mathrm{Ur}_2 = \frac{A_2\lambda_2^2}{h_2^3} = \left(\frac{h_1}{h_2}\right)^2. \tag{21}$$

Since the wave after the bottom step has a soliton-like shape (but it is not a soliton), the calculation of the amplitudes of the emerging solitons is relatively simple (for more details, see [22, 23]) and the formula for the secondary soliton amplitudes has the following form:

$$\frac{A_{n+1}}{A_0} = \frac{1}{4Ur_2}\left[\sqrt{1+8Ur_2} - (1+2n)\right]^2, \qquad n = 0, 1, 2, \dots N, \tag{22}$$

where N is the number of solitons, which is found as minimum positive value of the expression in square brackets in the relation (22). Particularly, the amplitude of the first (leading) soliton is

$$\frac{A_1}{A_0} = \frac{1}{4}\left(\frac{h_2}{h_1}\right)^2 \left[\sqrt{1+8\left(\frac{h_1}{h_2}\right)^2} - 1\right]^2 . \tag{23}$$

If after the bottom step the channel depth is very small ($h_2 << h_1$) and the width, respectively, is large, then the amplitude of the first soliton tends to two (and in the channel of constant cross-section it tends to four, see [22]). The number of solitons is rather high at that.

If the wave propagates towards the deep water ($h_2 >> h_1$), then

$$\frac{A_1}{A_0} \approx 4\left(\frac{h_1}{h_2}\right)^2, \qquad (h_2 >> h_1). \tag{24}$$

In this case the amplitude of soliton is small and only one soliton is formed. Dependence of generated soliton amplitude on depth changes (according to the expression (22)) is given in Fig. 2.

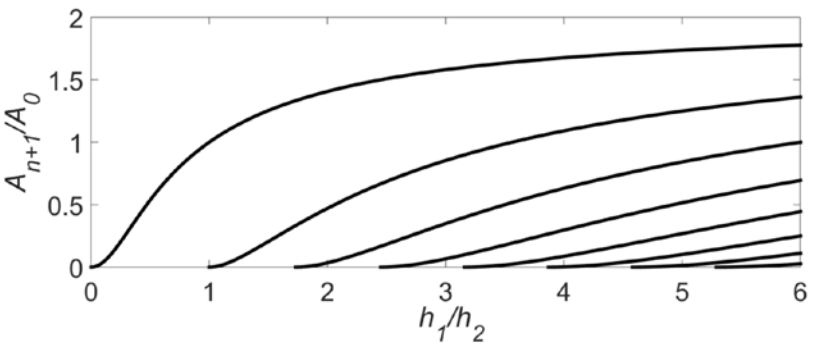

Fig. 2. Dependence of generated soliton amplitude on depth changes

Thus, at large distances even a weak nonlinearity and dispersion can significantly distort the wave process. In the self-consistent channel reflection is absent and all the effects of transformation are due to nonlinearity and dispersion only.

5. Conclusion

The given paper is a tribute to S.F. Dotsenko, who was a well-known expert in the field of water wave motion (including tsunamis). In recent years he and his team have published several works on the long wave channel theory. This subject is of special interest for the authors of this paper, and we present here a series of new solutions of the channel theory. First of all, this is the existence of traveling waves in a channel of variable cross-section within the framework of linear shallow water theory. Particularly, the so-called self-consistent channel, where Green's factor remains equal to one regardless the channel depth and width variations, is studied. In this case the wave does not change its shape during the propagation and inhomogeneity of the channel affects the wave propagation time only. The effect of nonlinearity and dispersion radically transforms the nature of the wave process at large distances. If the depth varies slowly, then the soliton changes adiabatically as it propagates. In this case, its amplitude and wavelength change, as well as a weak tail is generated behind. If the channel contains a bottom step, then when the wave propagates towards the shallow water behind the step, the initial perturbation disintegrates into solitons. Explicit formulas for the amplitudes of the generated solitons are given. The presented results attest to interesting features of wave dynamics in narrow bays and straits of variable cross-section.

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