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Velocity of the Wave Currents under the Floating Elastic Ice Formed by Nonlinear Interaction of the Wave Harmonics

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Purpose. The aim of the paper is to study dependence of the homogeneous fluid movement velocity (moving in the direction of wave propagation and formed by nonlinear interaction of the wave harmonics) upon the characteristics of the ice cover.
Methods and Results. Based on the movement velocity potential of the fluid of finite depth obtained in a form of an asymptotic expansion up to the values of the third order of smallness, analyzed was the velocity of fluid particles movement under the floating elastic ice at nonlinear interaction of the wave harmonics. Influence of the ice cover thickness and elasticity module, nonlinearity of the ice vertical acceleration, and the amplitude of the second interacting harmonic upon the components of the orbital velocity of the fluid particles movement under the floating ice was studied.
Conclusions. It is shown that the influence of nonlinearity of the vertical displacements' acceleration of floating ice upon the components of the fluid movement velocity is manifested in an increase of the phase shift. A change of a sign of the second interacting harmonic results in transformation of the profiles and decrease of the phase. Growth of the Young's modulus value is manifested in a noticeable increase of the phase shift and in a weak increase of the maximum values of the fluid movement velocity components as compared to the case when there is no ice.

Key words: nonlinear interaction of waves, flexural-gravitational waves, waves of finite amplitude, motion of fluid particles

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Introduction

Velocities of the progressive fluid displacement towards the motion of finite amplitude waves were studied in [1–3] at infinite, and in [4–6] at a finite depth of a basin with a free surface. In a linear formulation, the effect of floating broken ice on the velocity of wave currents in a homogeneous fluid was considered in [7]. Dependence of the components of the orbital velocity of motion of fluid particles with an open surface on the characteristics of a traveling periodic wave of finite amplitude was studied in [8], and under a floating elastic ice cover – in [9]. Experimental studies of the influence of the under-ice currents velocity on the parameters of flexural-gravitational waves are presented in [10].

In the present paper, based on the obtained solution to the problem of oscillations formed by nonlinear interaction of harmonics of progressive surface waves in the ice-fluid system [11], the dependence of the distribution of the components of the orbital velocity of homogeneous fluid particles motion along the wavelength of the formed wave on the ice cover characteristics was analyzed.

The results obtained can be used to interpret the results of laboratory and field observations, in the development of technologies and systems for monitoring sea basins during the ice period.

Problem statement

Consider a homogeneous ideal incompressible fluid of constant depth H. Its surface is covered with floating ice with a thickness h = const. Fluid and ice cover in horizontal directions is not limited. Let us study the effect of ice on the orbital velocities of fluid particles motion, formed by the interaction of two harmonics of finite amplitude waves. Let us assume that the fluid movement is potential, the ice oscillations are continuous, and the dimensionless variables $x = kx_1,\ z = kz_1,\ t = \sqrt{kg}\, t_1$, $\zeta = k\zeta^*$, $\varphi = \left(k^2/\sqrt{kg}\right)\varphi^*$, where k is the wave number; g is the acceleration of gravity; t is the time; $\varphi(x, z, t)$ is the fluid velocity potential, then the problem is to solve the Laplace equation

$$\varphi_{xx} + \varphi_{zz} = 0,\ -\infty < x < \infty,\ -H \le z \le \zeta \tag{1}$$

for the velocity potential with boundary conditions on the ice – liquid surface ($z = \zeta$)

$$D_1 k^4 \frac{\partial^4 \zeta}{\partial x^4} + \kappa k \frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2 - \frac{\partial \varphi}{\partial t}\right] = p\,, \tag{2}$$

$$p = \frac{\partial \varphi}{\partial t} - \zeta - \frac{1}{2}\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2\right]$$

and at the basin bottom ($z = -H$)

$$\frac{\partial \varphi}{\partial z} = 0. \tag{3}$$

At the initial moment of time ($t = 0$)

$$\zeta = f(x),\quad \frac{\partial \zeta}{\partial t} = 0\,. \tag{4}$$

Here

$$D_1 = \frac{D}{\rho g},\quad D = \frac{Eh^3}{12(1-\nu^2)},\quad \kappa = h\frac{\rho_1}{\rho}\,,$$

E, h, ρ_1 and ν are the modulus of normal elasticity, thickness, density and Poisson's ratio of ice, respectively; ρ is the fluid density; $\zeta(x, t)$ is the elevation of the ice – fluid surface, at the initial moment of time equal to the function $f(x)$. The velocity potential and the elevation of the ice – liquid surface at $z = \zeta$ are related by the kinematic condition

$$\frac{\partial \zeta}{\partial t} - \frac{\partial \zeta}{\partial x}\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial z} = 0\,. \tag{5}$$

In dynamic condition (2), the expression with the factor κ is the inertia of vertical ice displacements, where the first term in parentheses of this expression characterizes the nonlinearity of its vertical acceleration [11].

Expressions for the components of the orbital velocity of fluid particles motion

Solution of the problem (1) – (5) was found by the method of multiple scales [12] in the form of equations for three approximations, taking into account the nonlinearity of the acceleration of vertical displacements of elastic ice [11]. Consider periodic waves, specifying the function $f(x)$ in the appropriate form. For the case of interaction of traveling periodic waves, when the first approximation is given in the following form

$$\zeta_1 = \cos\theta + a_1 \cos 2\theta, \quad \theta = x + \tau T_0 + \beta(T_1, T_2),$$

where a_1 is the amplitude of the second interacting harmonic; $T_1 = \varepsilon t,\ T_2 = \varepsilon^2 t$, and $\beta(T_1,\ T_2)$ is found from the second and third approximations, the expression that determines the velocity potential up to the third approximation in dimensionless variables has the form as follows

$$\varphi = \varepsilon b_{11}\mathrm{ch}(z+H)\sin\theta + \sum_{n=1}^{3}\varepsilon^n b_{n2}\mathrm{ch}2(z+H)\sin 2\theta +$$

$$+\sum_{n=2}^{3}\varepsilon^n \sum_{j=3}^{4} b_{nj}\mathrm{ch}j(z+H)\sin j\theta + \varepsilon^3 \sum_{n=5}^{6} b_{3n}\mathrm{ch}n(z+H)\sin n\theta + \sum_{n=2}^{3}\varepsilon^n b_{n0} t,$$

$$\theta = x + \sigma t, \quad \sigma = \tau + \varepsilon\sigma_1 + \varepsilon^2\sigma_2, \ \varepsilon = ak.$$

Here

$$\tau^2 = \left(1 + D_1 k^4\right)\left(1 + \kappa k\,\mathrm{th}H\right)^{-1}\mathrm{th}H,$$

$$b_{11} = \frac{\tau}{\mathrm{sh}H},\ b_{12} = a_1\frac{\tau}{\mathrm{sh}2H},$$

$$b_{20} = \tau^2\left(a_1^2\left(1+\mathrm{cth}^2 2H\right) + \frac{1}{4}\left(1+\mathrm{cth}^2 H\right) + \kappa k\left(\frac{1}{2}\mathrm{cth}H + 4a_1^2\mathrm{cth}2H\right)\right),$$

$$b_{23} = \frac{l_3\mu_3 + 3l_7\tau}{3\mathrm{sh}3H\left(\mu_3 - 9\kappa k\tau^2 - 3\tau^2\mathrm{cth}3H\right)},$$

$$b_{24} = \frac{l_4\mu_4 + 4l_8\tau}{3\mathrm{sh}4H\left(\mu_4 - 16\kappa k\tau^2 - 4\tau^2\mathrm{cth}4H\right)},$$

$$b_{30} = a_1\tau^2\left(2\mathrm{cth}2H + \frac{1}{2}\mathrm{cth}H + \kappa k\left(\frac{9}{4} + 2\mathrm{cth}2H\mathrm{cth}H - \frac{1}{4}\mathrm{cth}^2 H\right)\right),$$

$$b_{3i} = \frac{j_i\mu_i + im_i\tau}{i\,\mathrm{sh}iH\left(\mu_i - i^2\tau^2\kappa k - i\tau^2\mathrm{cth}iH\right)},\ i = 3...6,\ \mu_i = 1 + i^4 D_1 k^4, i = 1...6,$$

$$a_{23} = \mu_3^{-1}\left(l_7 + 3\tau b_{23}\left(\mathrm{ch}3H - \kappa k 3\mathrm{sh}3H\right)\right),$$

$$a_{24} = \mu_4^{-1}\left(l_8 + 4\tau b_{24}\left(\mathrm{ch}4H - \kappa k 4\mathrm{sh}4H\right)\right),$$

$$a_{3i} = \left(i\tau b_{3i}\left(\mathrm{ch}iH + i\kappa k\,\mathrm{sh}iH\right) + m_i\right)\mu_i^{-1},\ i = 3...6\,,$$

$$a_1 = \pm\left(\frac{\mu_2 r_1}{4r_2\left(2\tau^2\mathrm{cth}2H + 4\tau^2\kappa k + \mu_2\right)\left(1 + 2\kappa k\,\mathrm{th}2H\right)}\right)^{1/2},$$

$$r_1 = \left(2\mathrm{cth}H + \mathrm{th}2H\left(\mathrm{cth}H\left(\frac{1}{2}\mathrm{cth}H + 3\kappa k\right) - \frac{5}{2}\right)\right)\left(\tau^2\left(\mathrm{cth}H + \kappa k\right) + \mu_1\right),$$

$$r_2 = \tau^2\left(\frac{1}{2} + \mathrm{cth}2H\,\mathrm{cth}H - \kappa k\left(\mathrm{cth}2H - \frac{5}{2}\mathrm{cth}H\right)\right) + \mu_1\left(\frac{1}{2}\mathrm{cth}H + \mathrm{cth}2H\right),$$

$$l_3 = -\frac{3}{2}a_1\tau\left(2\mathrm{cth}2H + \mathrm{cth}H\right),\quad l_4 = -4a_1^2\mathrm{cth}2H\,,$$

$$l_7 = a_1\tau^2\left(\frac{11}{2} - \mathrm{cth}2H\mathrm{cth}H + \kappa k\left(5\mathrm{cth}2H - \frac{1}{2}\mathrm{cth}H\right)\right),$$

$$l_8 = a_1^2\tau^2\left(5 - \mathrm{cth}^2 2H + 4\kappa k\,\mathrm{cth}2H\right),$$

$$j_3 = -\frac{5}{8}\tau - \frac{3}{8}a_1^2\tau - 6b_{24}\mathrm{ch}4H - \frac{3}{2}a_{24}\tau\mathrm{cth}H + 3a_{23}\sigma_1\,,$$

$$j_4 = -\frac{9}{2}a_1\tau - 6b_{23}\mathrm{ch}3H - 2a_{23}\tau\mathrm{cth}H + 4a_{24}\sigma_1\,,$$

$$j_5 = -\frac{69}{8}a_1^2\tau - 10b_{24}\mathrm{ch}4H - \frac{5}{2}a_{24}\tau\mathrm{cth}H - 5a_1\left(\frac{3}{2}b_{23}\mathrm{ch}3H - a_{23}\tau\mathrm{cth}2H\right),$$

$$j_6 = -5a_1^3\tau - 6a_1\left(2b_{24}\mathrm{ch}4H - a_{24}\tau\mathrm{cth}2H\right),$$

$$\sigma_1 = \frac{\tau\mu_2\left(2\mathrm{cth}H + \mathrm{th}2H\left(\mathrm{cth}H\left(\frac{1}{2}\mathrm{cth}H + 3\kappa k\right) - \frac{5}{2}\right)\right)}{4a_1\left(2\tau^2\mathrm{cth}2H + 4\tau^2\kappa k + \mu_2\right)\left(1 + 2\kappa k\,\mathrm{th}2H\right)},$$

$$\sigma_2 = \frac{1}{4}\left(\frac{q_1}{\mu_1} + \frac{q_2}{2a_1\mu_2}\right),$$

$$m_3=\tau\left(\frac{9}{2}a_1\sigma_1+2b_{24}\mathrm{ch}4H\left(2\mathrm{th}4H-\mathrm{cth}H\right)\right)+$$
$$+\frac{1}{2}\tau^2\left(\frac{1}{4}\mathrm{cth}H\left(1-23a_1^2\right)+7a_1^2\mathrm{cth}2H-3a_{24}\right)+3b_{23}\sigma_1\mathrm{ch}3H+$$
$$+\kappa k\left(\tau\left(2b_{24}\mathrm{sh}4H\left(11\mathrm{cth}4H-4\mathrm{cth}H\right)+3a_1\sigma_1\left(2\mathrm{cth}2H+\frac{1}{2}\mathrm{cth}H\right)\right)+\right.$$
$$\left.+\tau^2\left(a_1^2\left(\frac{21}{8}-2\mathrm{cth}^2 2H-\frac{7}{2}\mathrm{cth}H\,\mathrm{cth}2H\right)-\frac{1}{8}-\frac{3}{2}a_{24}\mathrm{cth}H-\frac{1}{2}\mathrm{cth}^2H\right)+9b_{23}\sigma_1\mathrm{sh}3H\right),$$

$$m_4=\tau\left(4\sigma_1a_1^2+\frac{3}{2}b_{23}\mathrm{ch}3H\left(5\mathrm{th}3H-\mathrm{cth}H\right)\right)+2\tau^2\left(a_1\mathrm{cth}2H-\frac{1}{4}a_1\mathrm{cth}H+a_{23}\right)+$$
$$+4b_{24}\sigma_1\mathrm{ch}4H+\kappa k\left(\tau\left(\frac{3}{2}b_{23}\,\mathrm{sh}3H\left(11\mathrm{cth}3H-3\mathrm{cth}H\right)+8a_1^2\sigma_1\mathrm{cth}2H\right)+\right.$$
$$\left.+\tau^2\left(a_1\left(\frac{37}{4}-4\mathrm{cth}2H\,\mathrm{cth}H-\frac{3}{4}\mathrm{cth}^2H\right)+2a_{23}\mathrm{cth}H\right)+16b_{24}\sigma_1\mathrm{sh}4H\right),$$

$$m_5=\tau\left(2b_{24}\mathrm{ch}4H\left(6\mathrm{th}4H-\mathrm{cth}H\right)+3b_{23}a_1\mathrm{ch}3H\left(\frac{7}{2}\mathrm{th}3H-\mathrm{cth}2H\right)\right)+$$
$$+\tau^2\left(\frac{7}{2}a_1^2\left(\mathrm{cth}2H-\frac{1}{4}\mathrm{cth}H\right)+5a_{23}a_1+\frac{5}{2}a_{24}\right)+$$
$$+\kappa k\left(\tau\left(2b_{24}\mathrm{sh}4H\left(19\mathrm{cth}4H-4\mathrm{cth}H\right)+3b_{23}a_1\mathrm{sh}3H\left(\frac{11}{2}\mathrm{cth}3H-3\mathrm{cth}2H\right)\right)+\right.$$
$$\left.+\tau^2\left(-a_1^2\left(\frac{3}{8}+6\mathrm{cth}^2 2H+\frac{11}{2}\mathrm{cth}2H\mathrm{cth}H\right)+10a_{23}a_1\mathrm{cth}2H+\frac{5}{2}a_{24}\mathrm{cth}H\right)\right),$$

$$m_6=4\tau b_{24}a_1\mathrm{ch}4H\left(4\mathrm{th}4H-\mathrm{cth}2H\right)+\tau^2a_1\left(a_1^2\mathrm{cth}2H+6a_{24}\right)+$$
$$+2\kappa ka_1\left(4\tau b_{24}\mathrm{sh}4H\left(5\mathrm{cth}4H-2\mathrm{cth}2H\right)+\tau^2\left(6a_{24}\mathrm{cth}2H-a_1^2\left(1+4\mathrm{cth}^2 2H\right)\right)\right),$$

$$q_1=\mu_1\left(\frac{3}{2}b_{23}a_1\mathrm{ch}3H-\tau\left(\frac{3}{8}-\frac{15}{4}a_1^2+a_{23}a_1\mathrm{cth}2H\right)\right)+\tau^2a_1\left(-\frac{1}{2}\sigma_1+\right.$$
$$\left.+3b_{23}\mathrm{ch}3H\left(\frac{1}{2}\mathrm{th}3H+\mathrm{cth}2H\right)\right)+\tau^3\left(9a_1^2\mathrm{cth}2H+a_1a_{23}+\frac{1}{4}\mathrm{cth}H\left(\frac{5}{2}-a_1^2\right)\right)+$$
$$+\kappa k\left(\tau^2a_1\left(3b_{23}\mathrm{sh}3H\left(\frac{1}{2}\mathrm{cth}3H+3\mathrm{cth}2H\right)-2\sigma_1\mathrm{cth}2H+\frac{1}{2}\sigma_1\mathrm{cth}H\right)+\right.$$
$$\left.+\tau^3\left(2a_{23}a_1\mathrm{cth}2H+\frac{1}{2}\mathrm{cth}^2H+\frac{3}{8}+a_1^2\left(8\mathrm{cth}^2 2H+\mathrm{cth}H\mathrm{cth}2H+\frac{39}{4}\right)\right)\right),$$

$$q_2 = \mu_2\left(3b_{23}\mathrm{ch}3H + 4b_{24}a_1\mathrm{ch}4H + \tau\left(a_{23}\mathrm{cth}H + 2a_{24}a_1\mathrm{cth}2H - 3a_1^3\right)\right) +$$
$$+ 2\tau^2\left(\frac{3}{2}b_{23}\mathrm{ch}3H\left(\mathrm{cth}H - \mathrm{th}3H\right) + 4b_{24}a_1\mathrm{cth}2H\mathrm{ch}4H - \sigma_1\right) +$$
$$+ 2\tau^3\left(a_{23} + a_1\left(3a_1\mathrm{cth}H + 2a_{24} + \mathrm{cth}2H\left(5a_1^2 - 2\right)\right)\right) +$$
$$+ \kappa k\left(2\tau^2\left(\frac{3}{2}b_{23}\mathrm{sh}3H\left(3\mathrm{cth}H - 5\mathrm{cth}3H\right) + 8b_{24}a_1\mathrm{sh}4H\left(2\mathrm{cth}2H - \mathrm{cth}4H\right) - \sigma_1\mathrm{cth}H\right) +\right.$$
$$\left. + 2\tau^3\left(2a_1^3\left(3 + 4\mathrm{cth}2H\right) + a_1\left(4\mathrm{cth}2H\left(a_{24} + \mathrm{cth}H\right) + \frac{1}{2}\mathrm{cth}^2H - \frac{3}{2}\right) + a_{23}\mathrm{cth}H\right)\right).$$

At that $b_{22} = b_{32} = a_2 = a_3 = l_1 = l_2 = l_5 = l_6 = j_1 = j_2 = m_1 = m_2 = 0$. In dimensional variables, the expression for the velocity potential is as follows:

$$\varphi = a\sqrt{g/k}\sum_{n=1}^{2} b_{1n}\,\mathrm{ch}\,nk(z+H)\sin n\theta + a^2\sqrt{kg}\left(\sum_{n=3}^{4} b_{2n}\,\mathrm{ch}\,nk(z+H)\sin n\theta + b_{20}t\right) +$$
$$+ a^3k\sqrt{kg}\left(\sum_{n=3}^{6} b_{3n}\,\mathrm{ch}\,nk(z+H)\sin n\theta + b_{30}t\right),$$

$$\theta = kx + \sqrt{kg}\left(\tau + ak\sigma_1 + a^2k^2\sigma_2\right)t,$$

And in the expressions for b_{20}, b_{30}, b_{11}, b_{12}, b_{23}, b_{24}, b_{33}, b_{34}, b_{35}, b_{36}, σ_1 and σ_2 the argument of hyperbolic functions is replaced by *kH*. Hereinafter, for expressions in dimensional variables, the symbols "x", "z", "t" are omitted from the index 1, and for "φ" – "*".

Thus, the horizontal ($u = \partial\varphi/\partial x$) and vertical ($w = \partial\varphi/\partial z$) components of the velocity of homogeneous fluid motion are determined by the expressions

$$u = a\sqrt{kg}\sum_{n=1}^{2} nb_{1n}\,\mathrm{ch}\,nk(z+H)\cos n\theta + a^2k\sqrt{kg}\sum_{n=3}^{4} nb_{2n}\,\mathrm{ch}\,nk(z+H)\cos n\theta +$$
$$+ a^3k^2\sqrt{kg}\sum_{n=3}^{6} nb_{3n}\,\mathrm{ch}\,nk(z+H)\cos n\theta,$$

$$w = a\sqrt{kg}\sum_{n=1}^{2} nb_{1n}\,\mathrm{sh}\,nk(z+H)\sin n\theta + a^2k\sqrt{kg}\sum_{n=3}^{4} nb_{2n}\,\mathrm{sh}\,nk(z+H)\sin n\theta +$$
$$+ a^3k^2\sqrt{kg}\sum_{n=3}^{6} nb_{3n}\,\mathrm{sh}\,nk(z+H)\sin n\theta.$$

Note that the obtained solution is valid outside small neighborhoods of the resonance values of the wave numbers k_i (i = 1...4), which are the positive real roots of the equations [11]

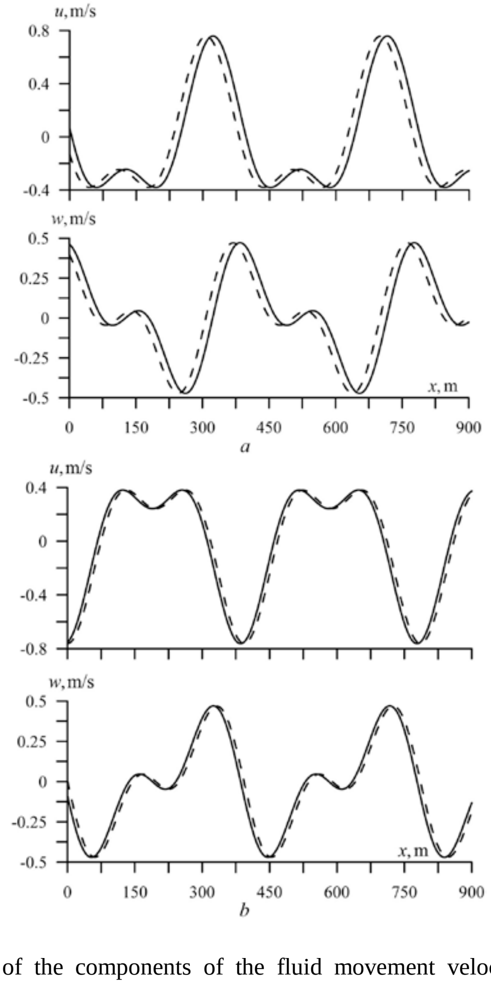
$$\mu_i - i^2\tau^2\kappa k - i\tau^2\mathrm{cth}iH = 0,\; i = 3...6. \qquad (6)$$

The left side of expression (6) is included in the denominator b_{3i}.

Analysis of the ice cover effect on the velocity components of fluid motion

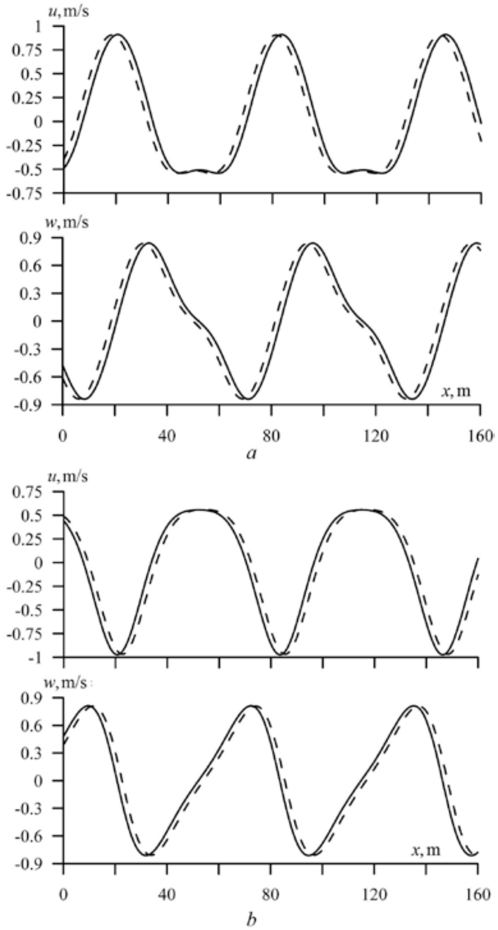
Evaluation of the influence of the floating ice characteristics on the velocity components in the direction of the nonlinear wave was carried out at $\rho_1/\rho = 0.87$, $\nu = 0.34$, $0 \le h \le 2$ m and E, equal to $0.5 \cdot 10^8$; 10^9; $3 \cdot 10^9$ N/m^2.

The u and w distributions along the wave profile are shown in Fig. 1 at $t = 3$ h, $a = 1$ m, $\lambda = 392.5$ m, $H = 45$ m, $h = 1$ m, $E = 3 \cdot 10^9$ N/m^2 with and without vertical ice acceleration. It can be seen that when the formed nonlinear wave propagates in the negative direction of the X axis, the effect of taking into account the nonlinearity of the acceleration of vertical displacements of ice on the fluid velocity components is manifested in the phase shift increase. A change in the sign of the second interacting harmonic from plus to minus leads to a noticeable transformation of the profiles and to a decrease in the phase (Fig. 1, *b*). The u and w profiles obtained taking in account the vertical acceleration nonlinearity lag behind the profiles obtained without it. The form of the generated perturbation is non-linear.



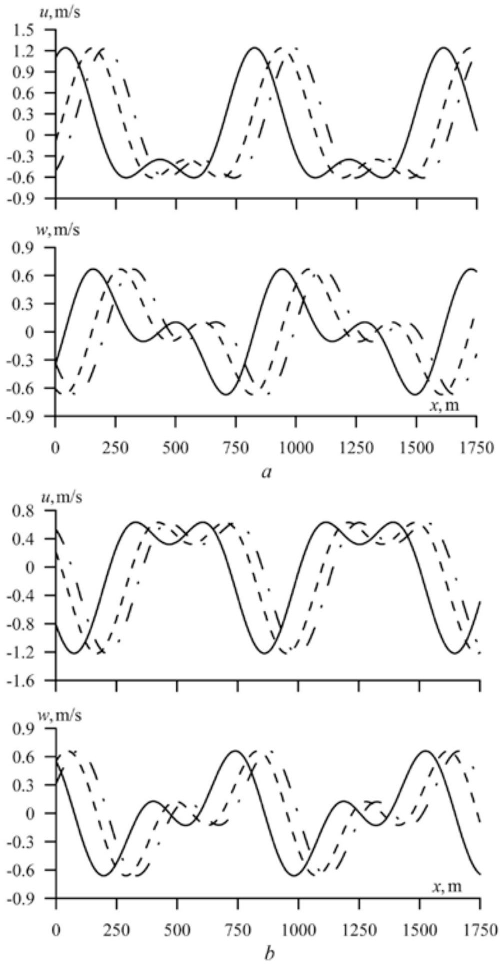
F i g. 1. Distribution of the components of the fluid movement velocity along the profile of a nonlinear wave at $a_1 > 0$ (*a*) and $a_1 < 0$ (*b*) at λ/H = 8.72 with the regard (dashed line) and with no regard (solid line) for the ice vertical acceleration

In the case of short waves (Fig. 2), the influence of vertical acceleration nonlinearity retains its direction, and the profile shape in the range of considered wave numbers remains nonlinear. Fig. 2 shows the distribution of the velocity components at $t = 540$ s, $a = 0.6$ m, $\lambda = 62.8$ m, $H = 70$ m, $h = 0.6$ m, $E = 3 \cdot 10^9$ N/m^2. For extreme values on the profile of the horizontal velocity component, as well as in the linear case and in the case of propagation of a periodic wave of finite amplitude [9], the values of the vertical component are equal to zero. At the same time, the extreme values on the vertical velocity component profile correspond to non-zero values of its horizontal component.



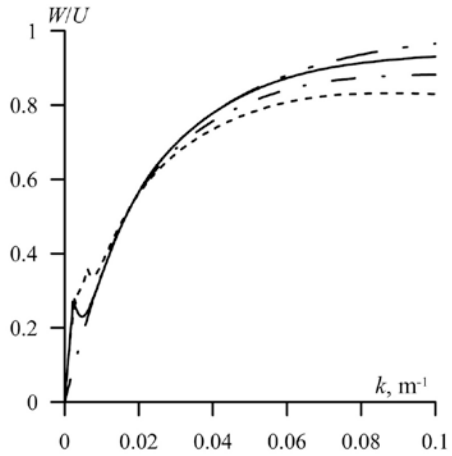
F i g. 2. Distribution of the components of the fluid movement velocity along the profile of a nonlinear wave at $a_1 > 0$ (*a*) and $a_1 < 0$ (*b*) at λ/H = 0.89 with the regard (dashed line) and with no regard (solid line) for the ice vertical acceleration

Influence of the elastic module of solid ice cover on the velocity components in the case of taking into account the vertical acceleration of ice is shown in Fig. 3. Here $t = 9900$ s, $a = 2$ m, $\lambda = 785$ m, $H = 70$ m, $h = 2$ m. The figure shows that an ice rigidity increase is manifested in a noticeable increase in the phase shift and a slight increase in the maximum values of the fluid velocity components. The sign change of a_1 from plus to minus deforms the profile both qualitatively and quantitatively (Fig. 3, *b*). In this case, an increase in ice rigidity, as in the case of $a_1 > 0$, leads to a noticeable increase in the phase with the regard to the phase shift when E = 0.

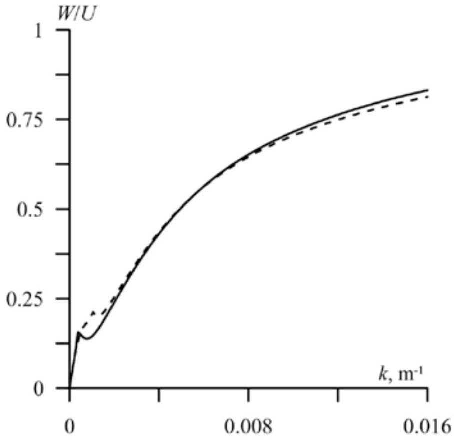


F i g. 3. Distribution of the components of the fluid movement velocity along the profile of a nonlinear wave with the regard for the ice vertical acceleration at $a_1 > 0$ (*a*) and $a_1 < 0$ (*b*). Solid line corresponds to the value $E = 3 \cdot 10^9$ N/m^2, dashed line – to $E = 10^9$ N/m^2 and dashed-dotted one – to $E = 0$

Ratio of the maximum values of the vertical velocity component (*W*) and the horizontal velocity component (*U*) for the case of broken ice ($h \neq 0$, $E = 0$), taking into account the vertical acceleration nonlinearity and the case when there is no ice (h = 0), is shown in Fig. 4. Here $a = 1$ m, $H = 30$ m, $E = 0$. Solid line – $h = 1$ m, $a_1 > 0$; dashed line – $h = 1$ m, $a_1 < 0$; dash-dotted line with two dots – $h = 0$, $a_1 > 0$; the dash-dotted line is a linear approximation at $h = 1$ m, $a_1 > 0$. From the analysis of the graphs, it follows that in the considered range of wave numbers, the distribution of *W/U* values over *k* is less than one; therefore, during the propagation of the formed nonlinear wave, the vertical velocity component does not exceed the horizontal one. This is also observed during the propagation of a periodic wave of finite amplitude [9]. In the region of small values of the wave numbers, the *W/U* change is not monotonous, excluding the linear approximation. With the *k* increase in the presence of ice, the values of the ratio of the velocity components decrease in comparison with the case of the ice cover absence on the fluid surface. At the same time, for the linear approximation, the *W/U* ratio values obtained for $a_1 > 0$ and h ≠ 0 are the smallest.



F i g. 4. Distribution of the *W*/*U* ratio value over the wave number at $E = 0$



F i g. 5. Distribution of the *W*/*U* ratio value over the wave number at $E = 3{\cdot}10^9$ N/m^2

Influence of the amplitude of the second interacting harmonic on the *W/U* ratio, taking into account the nonlinearity of the vertical acceleration of ice and $E \neq 0$, is shown in Fig. 5 for the long-wave range of wave numbers, where there are no resonant values [11]. Here $E = 3{\cdot}10^9$ N/m^2, $H = 100$ m, $a = 1$ m, $h = 1$ m. The solid and dashed lines correspond to the cases $a_1 > 0$ and $a_1 < 0$, respectively. It can be seen that the change in the a_1 sign is manifested both in an increase and decrease of the *W/U* ratio values. As *k* increases, the difference between the values grows. Note that in the shortwave range the W/U ratio at $E \neq 0$ and $a_1 > 0$ is greater than the W/U ratio at $E = 0$ and $a_1 > 0$, while at $E \neq 0$ and $a_1 < 0$ it is smaller than at $E = 0$ and $a_1 < 0$.

Conclusion

Based on the velocity potential of a finite depth fluid, obtained as an asymptotic expansion to values of the third order of smallness, the velocity of liquid particles under floating elastic ice is analyzed in the case of nonlinear interaction of wave harmonics. Influence of the ice cover thickness and elasticity module, the nonlinearity of the vertical acceleration of ice and the amplitude of the second interacting harmonic on the components of the orbital velocity of motion of fluid particles under floating ice is studied. Influence of floating broken ice on the velocity components is studied, and the case of propagation of a formed wave of finite amplitude in a basin with a free surface is also considered.

It is shown that the influence of the nonlinearity of the vertical displacements' acceleration of floating ice on the components of the fluid velocity is manifested in the phase shift increase. The sign reversal of the second interacting harmonic leads to a significant transformation of the profiles and a decrease in the phase. This effect is shown in the case of both short and long waves. Thus, neglecting the nonlinearity of the vertical acceleration of ice can lead to noticeable errors in determining the phase shift.

An increase in the value of the Young's modulus leads to an increase in the phase shift and an insignificant increase in the maximum values of the fluid velocity components with floating ice in comparison with the case when ice is absent. The phase also increases with an increase in the elastic ice thickness, and in the case of broken ice, it decreases.

Ratio of the maximum values of the vertical and horizontal velocity components in the considered range of wave numbers with an elastic module equal to zero is less than one. Consequently, during the propagation of a wave formed by the nonlinear interaction of wave harmonics, the vertical component of the velocity does not exceed the horizontal one. This is also observed during the propagation of a periodic wave of finite amplitude. A comparison of the W/U ratio distributions, obtained with and without the nonlinearity of the vertical acceleration of elastic ice, indicates its weak influence on the ratio. In this case, the change in the sign of the amplitude of the second interacting harmonic, both in the absence and in the presence of ice elasticity, has a noticeable effect.

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